COMS W4995-1: Intro to Social Networks	Due on Tuesday, November 1st, 2011

Assignment #4 – Infect

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Why there is two parts in this assignment: Each part fulfills one of the two objectives of the class:

- **Manipulate concepts:** Getting Familiar with the technical concepts used in class, by reproducing similar arguments. Being proficient by manipulating the object to answer some small-size problem.

  You are expected to answer this question rigorously, the answer can be quite short.
- **Connect the concepts to real-life:** Interpret a problem you find in light of the notions you have learned. Develop some critical eye w.r.t. how the concepts introduced are useful in practice.

How to read this assignment : Exercise levels are indicated as follows

($\rightarrowtail$) "elementary": the answer is not strictly speaking obvious, but it fits in a single sentence, and it is an immediate application of results covered in the lectures.

Use them as a <u>checkpoint</u>: it is strongly advised to go back to your notes if the answer to one of these questions does not come to you in a few minutes.

($\curvearrowright$) "intermediary": The answer to this question is not a simple application of results covered in class, it can be deduced from them with a reasonable effort.

Use them as a <u>practice</u>: how far are you from the answer? Do you still feel uncomfortable with some of the notions? which part could you complete quickly?

($\looparrowright$) "tortuous": this question either requires an advanced notion, a proof that is long or inventive, or it is still open.

Use them as an <u>inspiration</u>: can you answer any of them? does it bring you to another problem that you can answer or study further? It is recommended to work on this question only when you are done with the rest.

Part A
Practicing the concepts

Exercise 1: Continuous epidemics in the S↔I case

Motivation In this exercise, you will show that the final outcome in a epidemics with recovery to susceptible state depends on the parameter.

We consider a continuous (or fluid flow) model of an epidemic in a population, in the special case where nodes infect susceptible nodes with rate β and recover (*i.e.,* come back to the susceptible state) with rate γ. We denote by $y(t)$ the fraction of the population of nodes that are infected. Note that the fraction $x(t)$ of susceptible nodes satisfies $x(t) = 1 - y(t)$. According to our fluid flow assumption, both are real valued variables which increase and decrease continuously with time t.

Starting at time $t = 0$ with a fraction y_0, the evolution of this epidemic may be modeled as the solution of the following differential equation:

$$\frac{dy}{dt} = \beta x \cdot y - \gamma y = \beta(1 - y) \cdot y - \gamma y \ .$$

1 $(\curvearrowright)$ Assume that $\beta \neq \gamma$, find the solution of this differential equation for the initial condition $y(t) = y_0$ at time $t = 0$.

2 $(\curvearrowright)$ What is the limit behavior of the epidemics after a long time ($i.e.,$ characterized by $\lim_{t\to\infty} y(t)$) ? What is the role of y_0?

3 $(\curvearrowright)$ Assume now that $\beta = \gamma$, is the limit different from one of the two cases above? Can you compare how fast this limit is approached?

Part B

Concepts at large

Exercise 1:

As part of a health measurement campaigns, you are asked to derive from the results of epidemics in a population how likely it is to spread. In particular, you receive a very large number of population of viruses to classify into two categories:

- TYPE-I: a virus for which the human body becomes immune after going through a first infectious phase
- TYPE-II: a virus that is somewhat harder to recognize and need to be fought and recovered for every new infection, as if it was unknown.

Instead of running expensive clinical tests, you are asked to classify viruses only based on the dynamics that are observed, inside the homogeneous population of a city that we suppose relatively isolated from the rest of the world. You are given for different day the number of susceptible people, obtained through some complicated inference.

1. You are in particular observing that this number does not vanish with time but converge to a constant for most viruses. You are told that this probably means that all viruses are of TYPE-I in this case. What do you think of this statement? Which data would you need to distinguish the two cases?