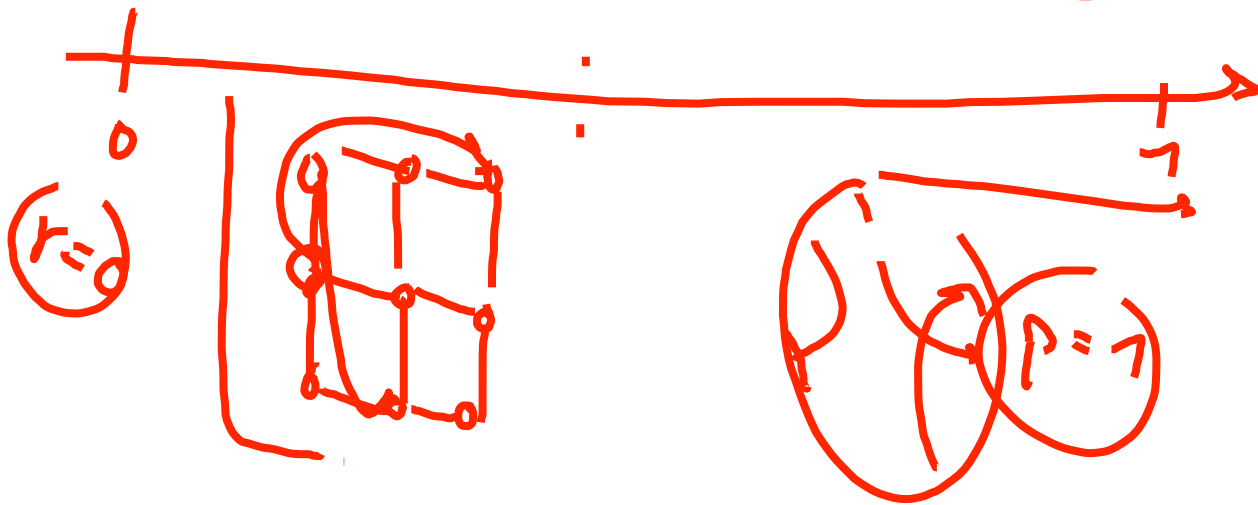
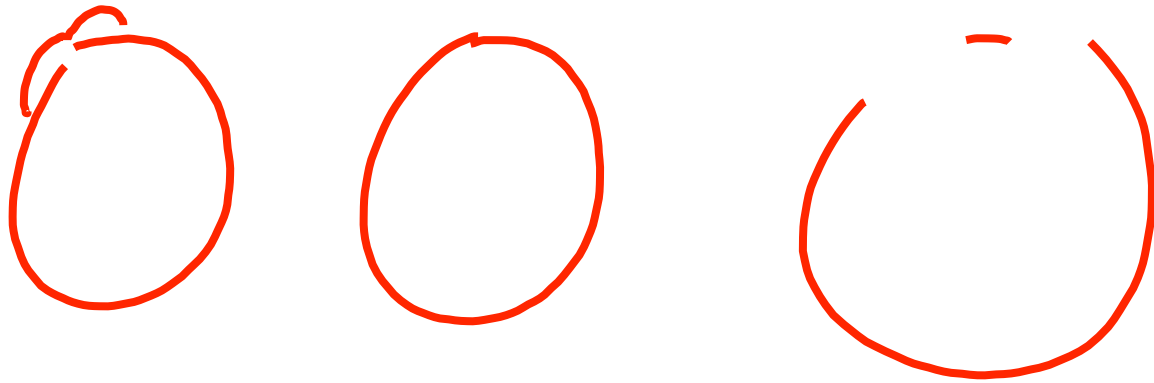


Correction

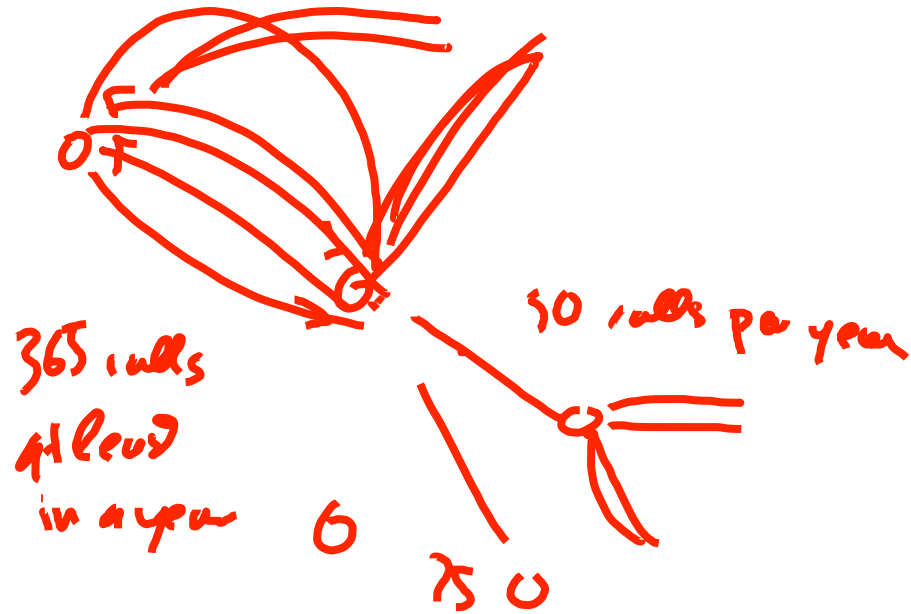
ASSIGNMENT #1

- Random graphs

ks 1 B-1



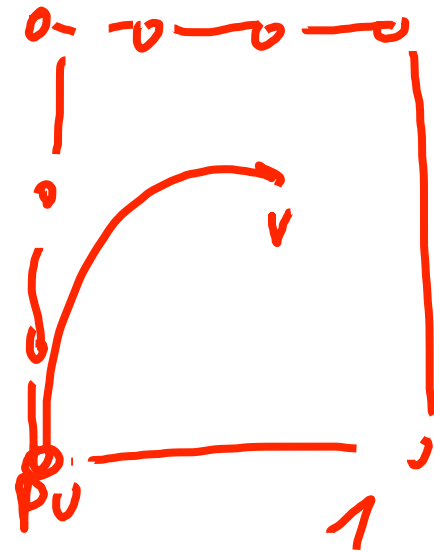
both gives  
long paths  
 $\gg \log N$



- needs his  
stray ties

no yard

Ass B.2.

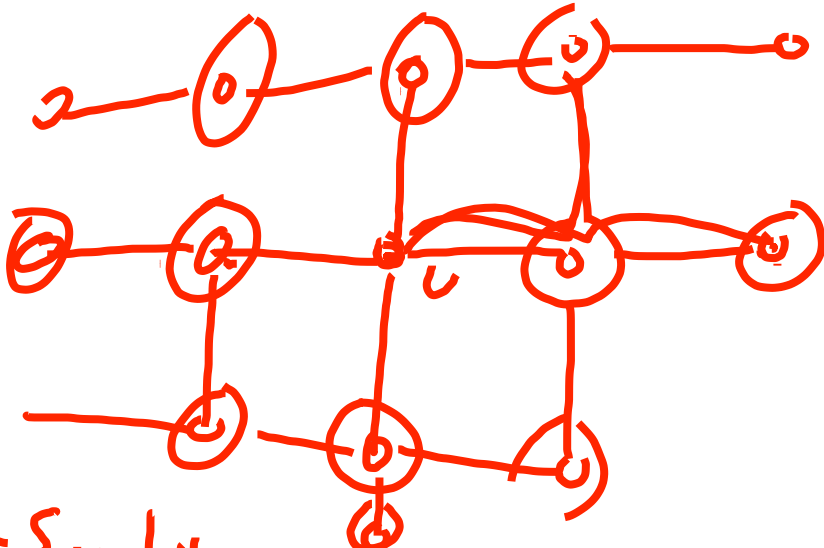


$$P(u \rightarrow v) = \frac{1}{\|u-v\|^r}$$

yard

# Part A

Ex 2:  $n=2$



Norms:

$$L_1: \|v-w\|_1 = |v_1 - w_1| + |v_2 - w_2|$$

$$L_2: \|v-w\|_2 = \sqrt{(v_1 - w_1)^2 + (v_2 - w_2)^2}$$

$$L_\infty: \|v-w\|_\infty = \max(|v_1 - w_1|, |v_2 - w_2|)$$

$$A_j = \{v \mid \|v - v_j\| \leq r_j\}$$

$$|A_j| = \binom{2j+1}{2}$$

$$\binom{2j+1}{2} \binom{2j+1}{2}$$

$$\max \{ |v_1 - w_1|, |v_2 - w_2| \} = j$$

$$L_\infty: \|v-w\| = \max$$

After choosing  $\|\cdot\|_\infty$  norm, For a given  $k \geq 0$

$$|A_j| = (2^{j+1})^k$$

$\exists \alpha, \beta > 0$  such that  $\alpha_j^k \leq |A_j| \leq \beta_j^k$  for  $j \geq 1$

$\exists \alpha > 0$  such that  $\alpha_j^k \leq (2^{j+1})^k$  e.g.  $\alpha = 2^k$  or

$\rightarrow \exists \beta > 0$   $\underline{\hspace{2cm}}$   $(2^{j+1})^k \leq \beta_j^k$  e.g.  $\beta = 3^k$   $\alpha \leq 2$

$$\beta = 3^k \quad \alpha \leq 2 \quad \alpha_j^k \leq 2^k j^k \leq (2^{j+1})^k$$

$$\underline{\hspace{1cm}} \quad (2^{j+1})^k \leq (\beta_j)^k \quad (2^{j+1})^k \geq (2^j)^k \geq 2^k j^k$$

$\|\cdot\|_1$ ,  $\exists \alpha, \beta$ , such that  $\forall x, \alpha \|x\|_1 \leq \|x\|_2 \leq \beta \|x\|_1$

---

$\exists \alpha, \beta$  such that  $\forall J, \alpha J^k \leq |\{v \mid \|u-v\|_1 \leq J\}| \leq \beta J^k$

$\forall J: 2^k J \leq |\{v \mid \|u-v\|_{\infty} \leq J\}| \leq 3^k J^k$

---

$\exists \alpha$  st.  $\forall J^k \leq |\{v \mid \|u-v\|_1 \leq J\}| \geq |\{v \mid \|u-v\|_{\infty} \leq J/\beta\}| \geq \alpha \frac{J^k}{\beta^k}$

$\|u-v\|_1 \leq J \iff \|u-v\|_{\infty} \leq J/\beta$

$\|u-v\|_{\infty} \leq p \Rightarrow \|u-v\|_1 \leq p \beta$

2)

2 Steps:  $\| \cdot \|_{\infty} \rightarrow \text{NO PR}$

$\nabla$  From  $\| \cdot \|_{\infty}$  we prove the result for  $\| \cdot \|_1$

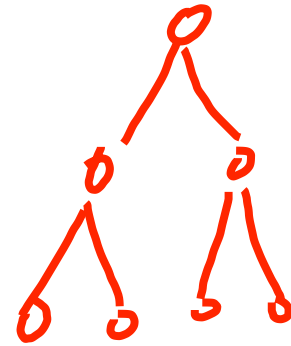
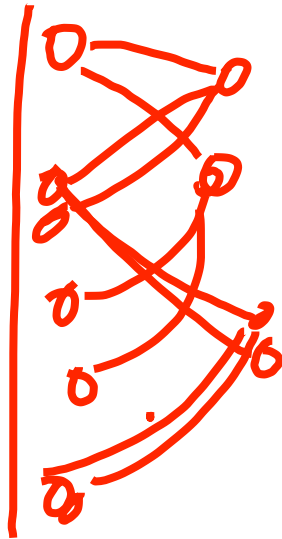
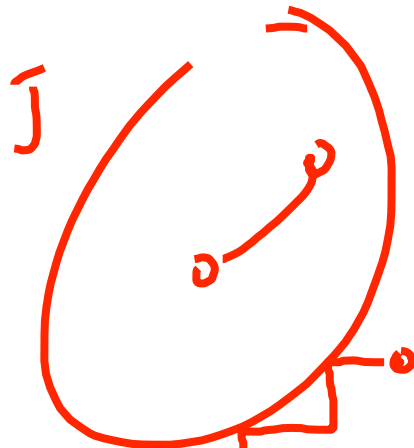
$$B_J = \left\{ v \mid \|v - v^j\| = r \right\} \quad B_J \in \Lambda_J$$

$$\exists \alpha, \beta \text{ such that } \forall j \geq 1 \quad \alpha j^{k-1} \leq B_J \leq \beta j^{k-1}$$

$$B_J = \left| \left\{ v \mid \max_{1 \leq i \leq k} |v_i| = j \right\} \right| = \alpha \times 2 \times (2j+1)^{k-1}$$

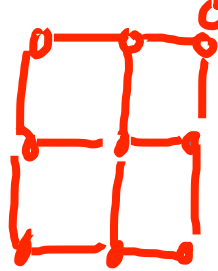
$\alpha, \beta, \dots, \alpha$

3



$\|u-v\|_{\mathcal{U}_J} = \# \text{ Steps in the grid}$

$A_J = \{ v \mid \text{reach from } u \text{ in } \leq J \text{ steps} \}$



$A_J = O(J^2)$

$A_J \propto J$

$A_J = \gamma J$



$$X_0(t+1) = \begin{cases} X_0(t) + 1 & \text{with prob. } 1 - p \frac{X_0(t)}{N(t)} \\ X_0(t) & \text{with prob. } p \frac{X_0(t)}{N(t)} \end{cases}$$

$$\cdot \frac{1 - X_0(t)}{N(t)}$$

$$+ 1 \quad \frac{1-p}{1-p + p \frac{1-X_0(t)}{N(t)}}$$

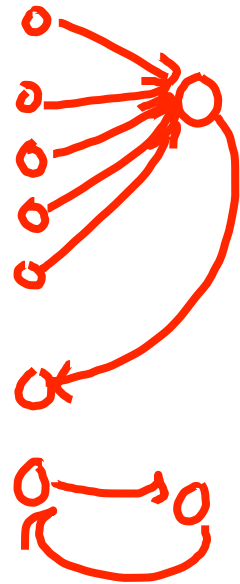
$$X_i(t+1) = \begin{cases} X_i(t) + 1 & \text{w.p. } p \frac{X_{i-1}(t)}{N(t)} + (1-p) \frac{(i-1)X_{i-1}(t)}{N(t)} \\ X_i(t) - 1 & \text{w.p. } p \frac{X_i(t)}{N(t)} + (1-p) \frac{X_i(t)}{N(t)} \\ X_i(t) & \text{otherwise} \end{cases}$$



$$c_i = \frac{1}{2i}$$

$$c_i = \frac{1}{2i}$$

$$c_i = \frac{1}{2i}$$



9

$$C_i = C_{i-1} \left( 1 - \frac{2-p}{(1+p) + i(1-p)} \right)$$

$$C_i = C_{i-1} \left( 1 - \frac{\beta}{i} + \varepsilon(i) \right) \text{ where } \beta = \frac{2-p}{1-p}$$

$$-\frac{2-p}{(1+p) + i(1-p)} + \frac{\beta}{i} = \varepsilon(i)$$

$$\varepsilon(i) \leq \frac{A}{i^2}$$







































