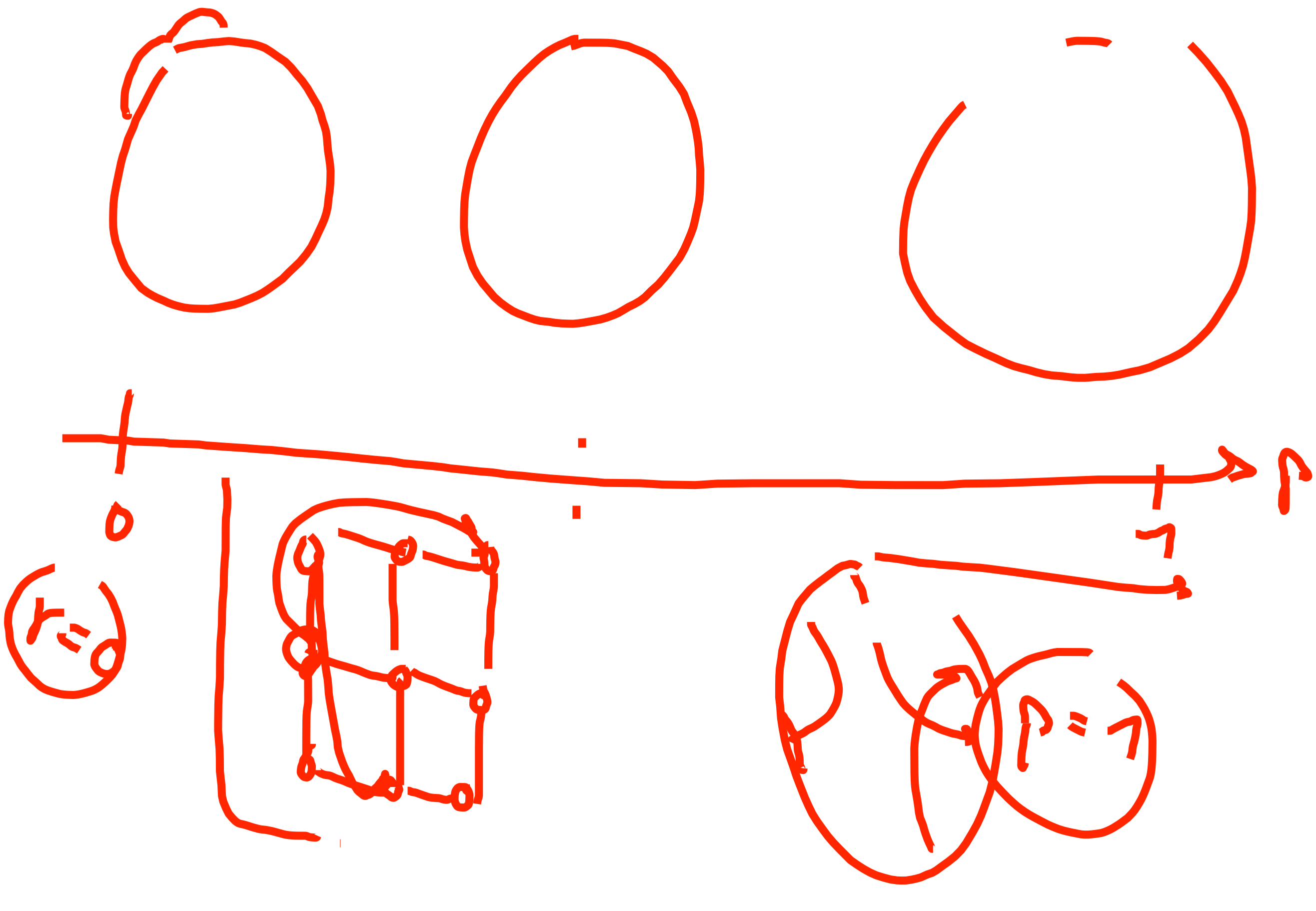
Correction

ASSIGNMENT #1

Lss1 B-1

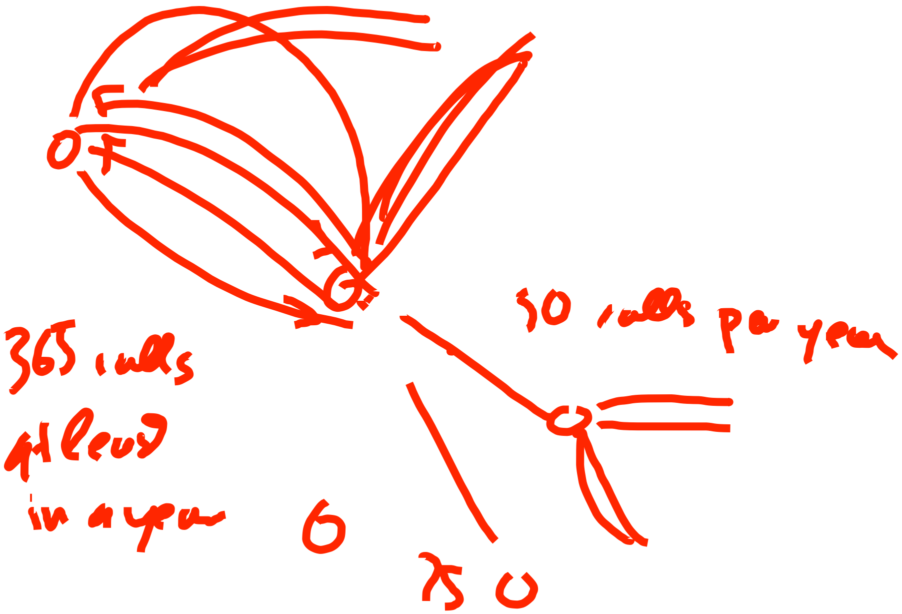
- Random graphs



Both gives long paths

$\gg \log N$

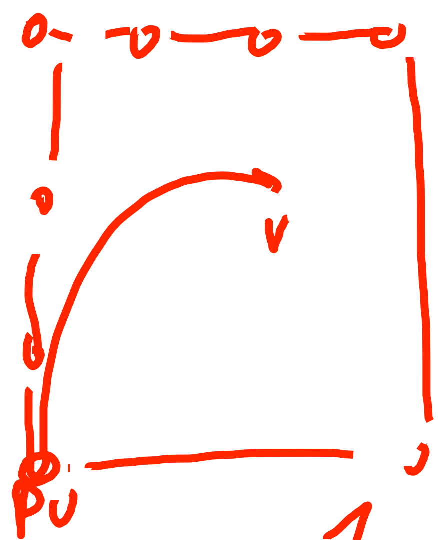
Ass B.2



- weak ties

strong ties

no grid



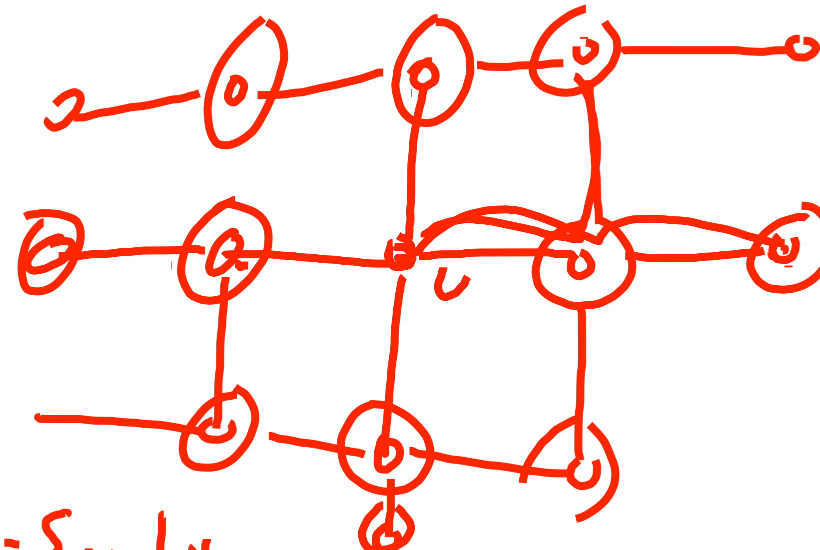
$$P(u \to v) = \frac{1}{\|u-v\|^r}$$

grid

Part A

Ex 2:

$k=2$



$A_j = \{ v \mid \|v - \cdot\|_\infty \le j \}$

$= 2$

$|A_j| = (2j+1)^k$

$= \{ v$

$(2j+1)(2j+1)$

$\max\{|v_1|, |v_2|, \ldots, |v_k|\} = j \}$

Norms:

L_1: $\|u-v\|_1 = |u_1 - v_1| + |u_2 - v_2|$

L_2: $\|u-v\|_{L_2} = \sqrt{(u_1-v_1)^2 + (u_2-v_2)^2}$

L_p: p $\quad \|u_1 - v_1|, |u_2 - v_2|$

L_∞: $\|u-v\| = \max$

After choosing L_∞ norm, for a given $k \geq 0$

$$|A_j| = (2j+1)^k$$

$\exists\, \alpha, \beta > 0$ such that $\alpha j^k \leq |A_j| \leq \beta j^k$ for $j \geq 1$

$\exists\, \alpha > 0$ such that $\alpha j^k \leq (2j+1)^k$ e.g. $\alpha = 2^k$ or $\alpha \leq 2$

$\rightarrow \exists\, \beta > 0$ ——— $(2j+1)^k \leq \beta j^k$ e.g. $\beta = 3^k$

$\alpha \leq 2$ $\quad \alpha j^k \leq 2 j^k \leq (2j+1)^k$

$\underline{\beta = 3^k}$ $\quad (2j+1)^k \leq (3j)^k \quad (2j+1)^k \geq (2j)^k \geq 2^k j^k$

$3^k j^k$

$\| \cdot \|_1$, $\exists\, \alpha_1, \beta_1$, such that $\forall x$, $\alpha_1 \|x\|_\infty \leq \|x\|_1 \leq \beta_1 \|x\|_\infty$

$\exists\, \alpha, \beta$ such that $\forall_J$ $\alpha J^k \leq \left| \{ v \mid \|u-v\|_1 \leq J \} \right| \leq \beta J^k$

$\forall J$: $\quad 2^k J \leq \left| \{ v \mid \|u-v\|_{L_\infty} \leq J \} \right| \leq 3^k J^k$

$\exists\, \alpha$ s.t. $\quad \alpha J^k \leq \left| \{ v \mid \|u-v\|_1 \leq J \} \right| \geq \left| \{ v \mid \|u-v\|_{L_\infty} \leq J/\beta_1 \} \right| \geq \alpha \dfrac{J^k}{\beta_1^k}$

$\|u-v\|_1 \leq J \quad \Longleftarrow \quad \|u-v\|_{L_\infty} \leq \dfrac{J}{\beta_1}$

$\|u-v\|_{L_\infty} \leq p \Rightarrow \|u-v\|_1 \leq p\, \beta_1$

2)

2 Steps: $\| \|_\infty \to$ NOPB

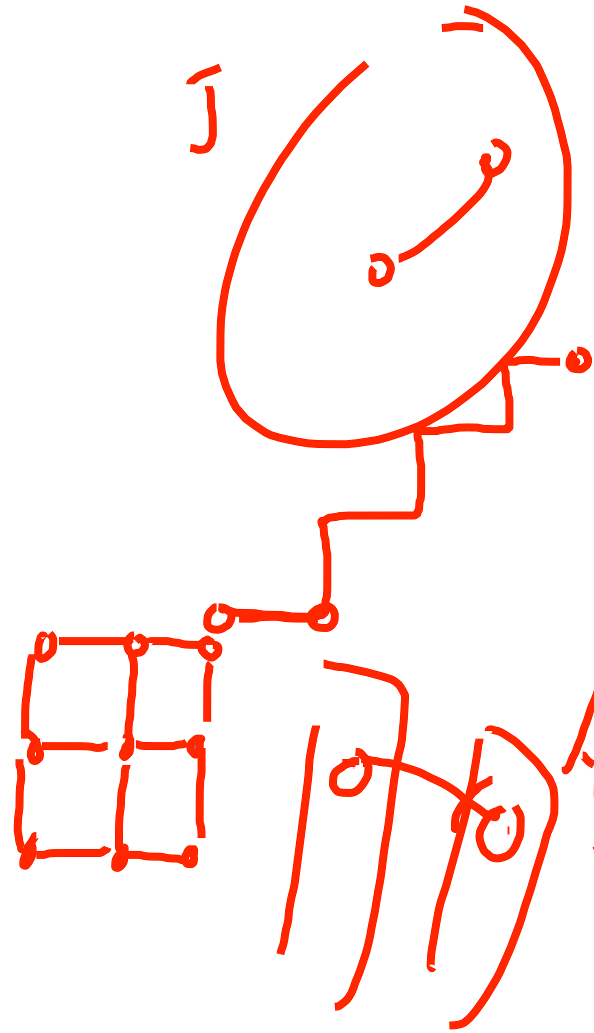
∇ From $\| \|_\infty$ we prove the result for $\| \|_1$

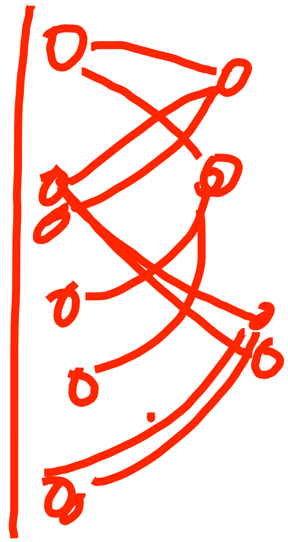
$$B_j = \left|\left\{ v \mid \|u-v\| = j \right\}\right| \qquad B_j \leq A_j$$

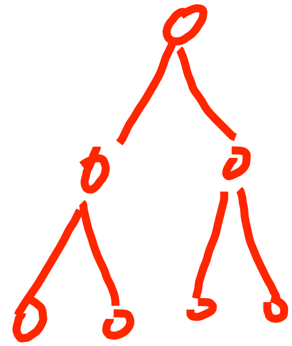
$\exists \alpha, \beta$ such that $\forall j \geq 1 \quad \alpha j^{k-1} \leq B_j \leq \beta j^{k-1}$

$$B_j = \left|\left\{ v \mid \max_{1,2,\dots,k} |v_1|, \dots, |v_k| = j \right\}\right| = k \times 2 \times (2j+1)^{k-1}$$

3







$\|u-v\|_{L^1}$ = # steps in the grid

$A_j = \{ v \mid$ reach from u in j steps $\}$

$A_j = O(j^d)$ $A_j \propto j$ $A_j = 2^j$

$$X_0(t+1) = \begin{cases} X_0(t)+1 & \text{wp} \quad 1-p\,\dfrac{X_0(t)}{N(t)} \\ \\ X_0(t) & p\,\dfrac{X_0(t)}{N(t)} \end{cases}$$

$$1-\frac{X_0(t)}{N(t)}$$

$$+1 \qquad 1-p$$

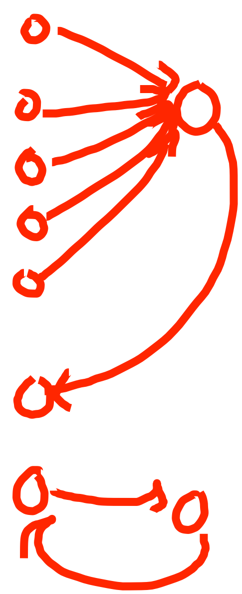
$$1-p+p\,\frac{1-X_0(t)}{N(t)}$$

$$X_i(t+1) = \begin{cases} X_i(t)+1 & \text{w.p. } p\,\frac{X_{i-1}(t)}{N(t)} + (1-p)\,\frac{(i-1)X_{i-1}(t)}{N(t)} \\ X_i(t)-1 & \text{w.p. } p\,\frac{X_i(t)}{N(t)} + (1-p)\,\frac{i X_i(t)}{N(t)} \\ X_i(t) & \text{otherwise} \end{cases}$$

$$c_i = \frac{c_{i-1}}{2}$$

$$c_i = \frac{1}{2^i}$$

$$c_i \to \frac{1}{i^{\alpha}}$$



9

$$C_i = C_{i-1}\left(1 - \frac{2-p}{(1+p)+i(1-p)}\right)$$

$$C_i = C_{i-1}\left(1 - \frac{\beta}{i} + \varepsilon(i)\right) \quad \text{where } \beta = \frac{2-p}{1-p}$$

$$-\frac{2-p}{(1+p)+i(1-p)} + \frac{\beta}{i} = \varepsilon(i) \qquad \varepsilon(i) \leq \frac{A}{i^2}$$